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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 22 July, 2021 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1 If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and B be a 3×3 matrix whose entries are {1,2,3,4,5}, then the number of possible matrices B such that AB = BA are :

(1) 240 (2) 320 (3) 120 (4) 100

Ans. **(3)**

Sol. $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let $B = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

$\Rightarrow \begin{bmatrix} p & q & r \\ a & b & c \\ x & y & z \end{bmatrix} = \begin{bmatrix} b & a & c \\ q & p & r \\ y & x & z \end{bmatrix}$

$\Rightarrow p = b$, $a = q$, $r = c$, $x = y$, & $z = z$

Hence number of such matrices are 5! = 120

2 The number of all possible numbers less than 10000 that can be formed using the digits 0, 2, 4, 6, 8

(without repetition) are :

Ans. **165**

Sol. 1 digit numbers = 5

2 digit numbers = $\underline{4}.\underline{4} = 16$

3 digit numbers = $\underline{4}.\underline{4}\,\underline{3} = 48$

4 digit numbers = $\underline{4}.\underline{4}\,\underline{3}\,\underline{2} = 96$

Total = 5 + 16 + 48 + 96 =165

intersection of lines $x - 2y = 4$ and $2x - y = 5$ also lies inside the circle, then the value of c lies in :

(1) $81 < c < 156$ (2) $100 < c < 156$ (3) $81 < c < 150$ (4) $100 < c < 150$

Ans. **(2)**

Sol. Intersection point of $2x - y = 5$ and $x - 2y = 4$ is $(2, -1)$

So, $(2, -1)$ lies indie the circle $\Rightarrow S_1 < 0$

$36(2)^2 + 36(-1)^2 - 108(2) + 120(-1) + c < 0$

$c < 156$ .....(i)

$\because$ circle $36x^2 + 36y^2 - 108x + 120y + c = 0$ neither touches nor cuts the co-ordinate axis so

$g^2 - c < 0 \Rightarrow \left(\frac{-3}{2}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 81$.....(ii)

and $f^2 - c < 0 \Rightarrow \left(\frac{5}{3}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 100$ ....(iii)

From (i), (ii) and (iii)

$100 < c < 156$

4. If line $2x + y = k$, $(k < 0)$ is a tangent to both the curves $x^2 - y^2 = 3$ and $y^2 = \alpha x$, then the value of α is

Ans. **24.00**

Sol. Given slope of line (m) = – 2

slope form of tangent to the curve $x^2 - y^2 = 3$ is $y = mx + \sqrt{a^2m^2 - b^2}$

On comparing, with the equation $2x + y = k$, $(k < 0) \Rightarrow k = -3$

Now, slope form of tangent to the parabola $y^2 = \alpha x$ is $y = mx + \frac{\alpha}{4m}$

But m = – 2 so

$y = -2x + \frac{\alpha}{4(-2)} \Rightarrow -3 = \frac{\alpha}{4 \times (-2)}$

$\alpha = 24$

5. The number of all possible values of $n \in \{1,2,3,\_\_\_\_ 100\}$ which satisfy the condition $11^n > 10^n + 9^n$ is:

Ans. (96)

Sol. Let $11^n > 10^n + 9^n$ $\quad n \in \{1,2,3,\_\_\_\_ 100\}$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow \frac{1}{5}[{}^nC_1 10^n + {}^nC_3 10^{n-2} + {}^nC_5 10^{n-4} + ...] > 10^n$

$\Rightarrow \frac{1}{5}[{}^nC_1 + {}^nC_3 10^{-2} + {}^nC_5 10^{-4} + ...] > 1$

Clearly the above inequality is true for $n > 5$

For n = 4 we have $\frac{1}{5}\left[4+\frac{4}{10^2}\right]=\frac{4}{5}\left(\frac{101}{100}\right)<1$, Rejected

Hence, number of such $n\in\{1,2,3,\_\_\_\_100\}$ is equal to 96

6. If an AP, $S_{10} = 530$ & $S_5 = 140$ (where S_n denotes the sum of first n terms of an AP), then the value of $S_{20} - S_6$ is

(1) 1562 (2) 1862 (3) 1762 (4) 1662

Ans. **(2)**

Sol. $S_{10} = 530$

$\frac{10}{2}[2a+9d]=530$

$2a + 9d = 106$ ... (1)

$S_5 = 140$

$\frac{5}{2}[2a+4d]=140$

$5d = 50$

$d = 10$

$a = 8$

Now,

$S_{20} - S_6 =$

$10[2a + 19d] - 3[2a + 5d]$

$14a + 175d$

$14 \times 8 + (175)10 = 1862$

7. The value of r for which the term independent of x in the expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180 is:

(1) 7 (2) 8 (3) 9 (4) 10

Ans. (2)

Sol. $T_{k+1} = {}^{10}C_k\,(2x^r)^{10-k}\,(x)^{-2k} \Rightarrow {}^{10}C_k\,2^{(10-k)}.\,x^{10r-rk-2k}$

Now, $10r - rk - 2k = 0 \Rightarrow r = \frac{2k}{10-k}$

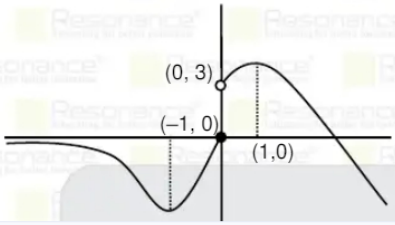
$r = \frac{2\times 8}{10-8} = 8$

8. If $f(x)=\begin{cases}-\frac{4}{3}x^3+2x^2+3 ; & x>0\\ 3xe^x ; & x\le 0\end{cases}$ then interval in which f(x) is increasing, is :

(1) $\left(-1,\frac{3}{2}\right)$ (2) (0, 1) (3) $\left(\frac{-3}{2},1\right)$ (4) $\left(\frac{1}{2},2\right)$

Ans. (B)

Sol. $f'(x)=\begin{cases}-4x^2+4x & , \ x>0\\ 3(x.e^x+e^x) & , \ x\le 0\end{cases}$



9. If $z^2+3\bar{z}=0$ has n solutions, then the value of $\sum_{k=0}^{\infty}\frac{1}{n^k}$ is :

(1) $\frac{3}{4}$ (2) $\frac{4}{3}$ (3) $\frac{5}{5}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. Let z = x + iy

$(x + iy)^2 + 3(x – iy) = 0$

$x^2 – y^2 + 2ixy + 3x – 3iy = 0$

$x^2 – y^2 + 3x = 0$ & $2xy – 3y = 0$

Case-1: y = 0

$x^2 – y^2 + 3x = 0$

$\Rightarrow$ x = 0 or x = –3

Solutions are z=0 and z=–3

Case-2: $x = \frac{3}{2}$

$x^2 – y^2 + 3x = 0$

$\Rightarrow y=\frac{3\sqrt{3}}{2}$ or $y=\frac{-3\sqrt{3}}{2}$

Solutions are $z=\frac{3}{2}+i\frac{3\sqrt{3}}{2}$ and $z=\frac{3}{2}-i\frac{3\sqrt{3}}{2}$

Total number of solutions = n = 4

So $\sum_{k=0}^{\infty}\frac{1}{4^k}=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$ Ans.

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10 The number of solutions of the equation $\sin^7 x + \cos^7 x = 1$ in the interval $[0, 4\pi]$ is :

(1) 7 (2) 9 (3) 14 (4) 5

Ans. (4)

Sol. $\sin^2 x + \cos^2 x = 1$, $\sin^2 x \le 1$ and $\cos^2 x \le 1$

$\sin^7 x \le \sin^2 x$

$\cos^7 x \le \cos^2 x$

so, $\sin^7 x + \cos^7 x \le 1$

$\sin^7 x + \cos^7 x = 1$ when $\sin^7 x = \sin^2 x$ & $\cos^7 x = \cos^2 x$

Case-2 : $\sin x = 1, \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}$

Total number of solutions = 5

11. The sum of all natural numbers belonging to the set {1, 2, 3 ..... 100}, whose HCF with 2304 is 1, is

(1) 2449 (2) 1633 (3) 1449 (4) 2633

Ans. (2)

Sol. $2304 = 2^8 \cdot 3^2$

Hence n can not be multiple of 2 or 3

Then sum is

$\Rightarrow n(1) - (n(2) + n(3) - n(6))$

(where n(a) means the sum of all numbers belonging to the set {1, 2, 3 .....100},which are divisible by a)

$$\Rightarrow \frac{100\times101}{2} - \frac{2\times50\times51}{2} - 3\times\frac{33\times34}{2} + 6\times\frac{17\times16}{2}$$

12. If f(x) is a continuous function defined as $f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \ln\left(\frac{1+\alpha x e^x}{(1+xe^x)^2}\right) & ; \ x<0 \\ \alpha & ; \ x\geq 0 \end{cases}$,then the value of α is

(1) $\frac{-2}{3}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{-1}{3}$

Ans. (1)

Sol. $= \lim\limits_{x\to0^-} \frac{x^3}{(1-\cos2x)^2} \ln\left(\frac{1+\alpha xe^x}{(1+xe^x)^2}\right)$

$$= \lim_{x\to0^-} \frac{x^3\times x}{4\sin^4 x} \frac{\ln(1+\alpha xe^x) - 2\ln(1+xe^x)}{x}$$

$$= \lim_{x\to0^-} \frac{1}{4} \frac{\ln(1+\alpha xe^x) - 2\ln(1+xe^x)}{x}$$

$$= \lim_{x\to0^-} \frac{1}{4}\left\{\frac{\ell n(1+\alpha xe^x)\alpha e^x}{\alpha e^x} - \frac{2\ell n(1+xe^x)}{\alpha e^x}e^x\right\}$$

$$= \frac{1}{4}(\alpha - 2)$$

Now, $\lim\limits_{x\to0^+} \alpha = \alpha$

$\because$ f(x) is continuous function so

$$\frac{\alpha-2}{4} = \alpha \Rightarrow \alpha = \frac{-2}{3}$$

13 If the domain of $f(x) = \frac{\cos^{-1}\sqrt{x^2-x+1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is $(\alpha + \beta]$, then the value of $\alpha + \beta$ is :

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) 1 (4) 2

Ans. (2)

Sol. $0 < x^2 - x + 1 < 1$ and $0 < \frac{2x-1}{x} < 1$

$\Rightarrow$ $x(x-1) \le 0$ & $-1 < 2x \le 3$

$\Rightarrow$ $x \in [0,1] \cap x \in \left(\frac{1}{2}, \frac{3}{2}\right]$

$\Rightarrow$ $x \in \left(\frac{1}{2}, 1\right]$

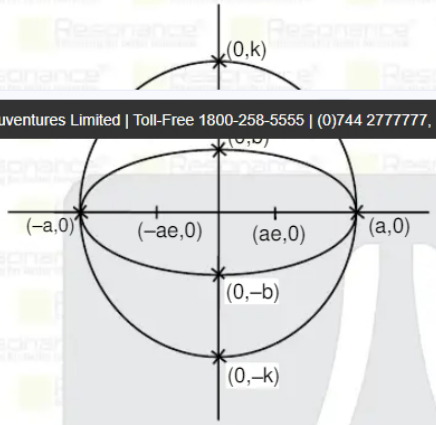
Hence $\alpha + \beta = \frac{1}{2} + 1 = \frac{3}{2}$

14. Let E_1 and E_2 be two Ellipses with same eccentricity. Where $E_1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b) and E_2 is such that it passes through the end points of major axis of E_1 also end points of minor axis of E_1 are foci of E_2. Find the eccentricity of E_1

(1) $\sqrt{\frac{\sqrt{5}-3}{2}}$ (2) $\sqrt{\frac{\sqrt{5}+3}{2}}$ (3) $\frac{\sqrt{5}+1}{2}$ (4) $\frac{\sqrt{5}-1}{2}$

Ans. (4)

Sol.



Eccentricity of E_1 is e $\Rightarrow$ $e^2 = 1 - \frac{b^2}{a^2}$

Eccentricity of E_2 is e $\Rightarrow$ $e^2 = 1 - \frac{a^2}{k^2}$

So, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{a^2}{k^2} \Rightarrow k = \frac{a^2}{b}$ .....(i)

From equation (i) and (ii) $e = \frac{b^2}{a^2}$

Since $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - e$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{\sqrt{5}-1}{2}$

15. Value of x for which $[e^x]^2 + [e^x + 1] - 3 = 0$ is

(1) $[e, e^2]$ (2) $[0, e]$ (3) $[e, \ell n 2]$ (4) $[0, \ell n 2]$

Ans. (4)

Sol. $[e^x]^2 + [e^x+1] - 3 = 0$

$[e^x]^2 + [e^x] - 2 = 0$

Let $[e^x] = t$

$t^2 + t - 2 = 0$

$(t+2)(t-1) = 0$

$t = 1, -2$

$x \in [0, \ell n 2]$

16. Evaluates $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$

(1) $e^x\sqrt{\frac{1+x}{1-2x}} + C$ (2) $e^x\sqrt{\frac{1+x}{1-x}} + C$ (3) $e^x\sqrt{\frac{1-x}{1+x}} + C$ (4) $e^x\left(\frac{1+x}{1-x}\right) + C$

Ans. **(2)**

Sol. $I = \int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}dx$

$= \int e^x \left\{ \frac{1}{(1-x)^{3/2}(1+x)^{1/2}} + \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} \right\} dx$

$\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$ $\frac{1}{}$

$\Rightarrow I = \int e^x (f'(x) + f(x))dx$

$= e^x f(x) + C = e^x\sqrt{\frac{1+x}{1-x}} + C$

17. Four dice are rolled and the outcomes are put in 2 × 2 matrices. Find the probability that such a matrix will be non singular and all its entries are different.

(1) $\frac{71}{81}$ (2) $\frac{80}{81}$ (3) $\frac{30}{71}$ (4) $\frac{25}{71}$

Ans. **(2)**

Sol. $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|x| = ad - bc = 0$

$\left.\begin{matrix} (1,6) & (3,2) \\ (3,4) & (6,2) \end{matrix}\right\} 8 + 8$ possibilites

18. Find the values of λ & μ for which the system of equations

$x + y + z = 6$

$x + 2y + \lambda z = \mu$ has no solution

(1) $\lambda = 2, \mu \neq 10$ (2) $\lambda \neq 2, \mu = 10$ (3) $\lambda \neq 3, \mu = 10$ (4) $\lambda \neq 2, \mu \neq 10$

Ans. **(1)**

Sol. For no solution $\Delta = 0$

$\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 1(5\lambda - 10) - 1(3\lambda - 5) + 1(6 - 5) = 0$

$\Rightarrow 2\lambda - 4 = 0$

$\Rightarrow \lambda = 2$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 26 & 5 & 5 \\ \mu & 2 & 2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & \mu & 2 \end{vmatrix}$$

$= 52 - 5\mu - 6 + 3\mu - 26$

$\Delta_2 = 20 - 2\mu$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 3 & 5 & 26 \\ 1 & 2 & \mu \end{vmatrix} = 1(5\mu - 52) - 1(3\mu - 26) + 6(6 - 5)$$

$\Delta_3 = 2\mu - 20$

Case-I

$\lambda = 2, \mu = 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

system of equations are

$x + y + z = 6$

$3x + 5y + 5z = 26$

$x + 2y + 2z = 10$ has infinite many solutions

Case - II

system has not solution

19. The number of elements in the set $\{x \in R : (|x| - 3)|x - 4| = 6\}$ is equal to

Ans. **(3)**

Sol. $(|x| - 3)|x - 4| = 6$

Case - 1

$x \geq 4$

$(x - 3)(x - 4) = 6$

$x^2 - 7x + 6 = 0$

$(x - 1)(x - 6) = 0$

$x = 1, 6 \Rightarrow x = 6$

Case – 2

$0 < x < 4$

$(x - 3)(4 - x) = 6$

$x^2 - 7x + 18 = 0$

$D < 0$, No solution

Case -3

$(x + 3)(x - 4) = 6$

$x^2 - x - 18 = 0$

$x = \frac{1 \pm \sqrt{73}}{2}$

$x = \frac{1 - \sqrt{73}}{2}$

20. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1 + 4\pi^2}$, $\alpha \in R$ where [x] is greatest integer function, then α is

(1) $50(e - 1)$ (2) $150(e^{-1} - 1)$ (3) $200(1 - e^{-1})$ (4) $100(1 - e)$

Ans. **(3)**

Sol. $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}} dx$

$\Rightarrow 50\int_0^{\pi} e^{-x/\pi}[1 - \cos 2x]dx$

$\Rightarrow 50\left[e^{-x/\pi} \times (-\pi)\right]_0^{\pi} - 50\int_0^{\pi} e^{-x/\pi} \cos 2x dx$

$\Rightarrow 50 \times (-\pi)(e^{-1} - 1) - \frac{50 \times \left[e^{-x/\pi}\left(\frac{-1}{\pi} \times \cos 2x + 2\sin 2x\right)\right]_0^{\pi}}{\left(\frac{1}{\pi^2} + 4\right)}$

$\Rightarrow -50\pi(e^{-1} - 1) - \frac{50\pi^2}{(1 + 4\pi^2)}\left[e^{-1}\left(\frac{-1}{\pi}\right) + \frac{1}{\pi}\right]$

$\Rightarrow \frac{200\pi^3(1 - e^{-1})}{1 + 4\pi^2}$ So $\alpha = 200(1 - e^{-1})$